Indian Statistical Institute
Bangalore Centre
B.Math (Hons.) Second Year 2009-2010
Second Semester
Backpaper Examination
Statistics II

Date : Time :- 3 hours

Answer as much as you can. The maximum you can score is 100
The notation used have their usual meaning unless stated otherwise.
State clearly the results that you assume.

1. (a) Define a sufficient statistics for a parameter. State factorization theorem.

   (b) Suppose $X_1, X_2, \cdots X_n$ is a random sample from a $N(\mu, \sigma^2)$ population. Show that $T(X) = (T_1 = \bar{X}, T_2 = \sum_{i=1}^{n} X_i^2)$ is jointly sufficient for (μ, σ^2).

   (c) When is a statistic called minimal sufficient ?

   Suppose $f(\underset{\sim}{x}, \theta)$ is the pdf of a sample $\underset{\sim}{X} = (X_1, X_2, \cdots X_n)$. Suppose there exists a function $T(\underset{\sim}{X})$ such that, for every two sample points $\underset{\sim}{x}$ and $\underset{\sim}{y}$, the ratio $f(\underset{\sim}{x}, \theta)/f(\underset{\sim}{x}, \theta)$ is independent of θ if and only if $T(\underset{\sim}{x}) = T(\underset{\sim}{x})$. Then show that $T(\underset{\sim}{X})$ is a minimal sufficient statistic for θ.

   Is $T(X)$ of question (b) minimal sufficient for (μ, σ^2) ? Give reason.

   (d) Define (a) ancillary statistic and (b) a complete statistic.

   (e) Suppose $X_1, X_2, \cdots X_n$ is a random sample from a uniform $(\theta, \theta{+}1)$ population. Let $T_1(\underset{\sim}{X}) = X_{(1)}$ and $T_2(\underset{\sim}{X}) = X_{(n)}$. Find the joint density of $T = (T_1, T_2)$. Find an ancillary statistic which is a function of T. What can you say about the completeness of T ?

   [(3+3)+ 4 + (3+5+3) + (3+3) + (6+4+2) = 39]

2. (a) Suppose $X_1, X_2, \cdots X_n$ is a random sample from Poisson (λ). Find the maximum likelihood estimate (M.L.E.) of λ.

   (b) If $\hat{\theta}$ is an M.L.E. of θ, then show that $h(\hat{\theta})$ is also an M.L.E. of $h(\theta)$ for any function h.

   [4 + 6 = 10]

3. (a) Formulate each of the following problems as a testing of hypothesis problems and suggest a test procedure, using a statistics with tabulated distribution functions whenever possible. Suggest how to collect data and how to handle the data successfully to test the hypothesis. State your assumptions clearly.

   (i) A customer wanted to buy an equipment. She wanted to be sure enough that the equipment would last for at least 6 months. [Hint : Assume that lifetime follows exponential distribution.]

   (ii) A company selling soft drink bought a machine for filling the bottles. The owner wanted to make sure that the amount of drink poured into a bottle does not vary too much.

   (iii) A shopkeeper wanted to see whether it was worth to keep the shop open after 10 P.M. [He would close down by 11 P.M. anyway]. He decided that he would not bother about 5 or less number of customers.

   (iv) A company wants to buy a bulk of items from a factory. The owner wanted to make sure that the proportion of defective items is not more than 5%.

(b) Show that the following families have a monotone likelihood ratio.

(i) Binomial(n, θ) with n known. (ii) Poisson (θ) and (iii) Exponential (θ).

(c) Using the information derived in Q(b), find UMP tests for the questions (i) and (iv) using $\alpha = 0.05$. State clearly the result you use.

[4x4 + 4x3 + 4x2 = 36]

4. Define uniformly most accurate (UMA) lower confidence bound with confidence level $1 - \alpha$ of a parameter θ. Suppose X is a continuous random variable having p.d.f $f(x, \theta)$. Suppose we want to find an UMA lower confidence bound with confidence level 0.95. Show how it is helpful to know an uniformly most powerful test of level 0.05 for a certain testing problem.

[4 + 10 =14]